$P(x) \quad x \in \mathbb{R}$

$P(X) \quad X \in \mathbb{R}^d$

$P(x,y) \sim (x^1, y^1), (x^2, y^2), \dots, (x^m, y^m)$

$p(x,y) = p(x)\, p(y|x)$

$p(x) = \sum_y p(x,y)$

$p(y|x) = \frac{p(x,y)}{p(x)}$

$x^i \sim p(x)$

$y^i \sim p(y|x^i)$

claim: (x^i, y^i) is a sample from $p(x,y)$

proof: $\Pr(X = x^i, Y = y^i) = p(x^i)\, p(y^i|x^i) = p(x^i, y^i)$

$\Rightarrow (x^i, y^i)$ is a sample drawn from $p(x,y)$

$p(x,y) = p(x)\, p(y|x)$

$p(x,y) = p(y)\, p(x|y)$

$p(x_1, x_2, \dots, x_n) = p(x_n | x_1 \text{---} x_{n-1})\, p(x_1 \text{---} x_{n-1})$

$= p(x_n | x_1 \text{---} x_{n-1})\, p(x_{n-1} | x_1 \text{---} x_{n-2})\, p(x_1 \text{---} x_{n-2})$

chain Rule

$= p(x_1)\, p(x_2|x_1)\, p(x_3|x_1, x_2)\, p(x_4|x_1, x_2, x_3) \dots p(x_n | x_1, x_2, \dots, x_{n-1})$

$x_1^i \sim p(x_1)$

$x_2^i \sim p(x_2 | x_1^i)$

$x_3^i \sim p(x_3 | x_1^i, x_2^i)$

$x_4^i \sim p(x_4 | x_1^i, x_2^i, x_3^i)$

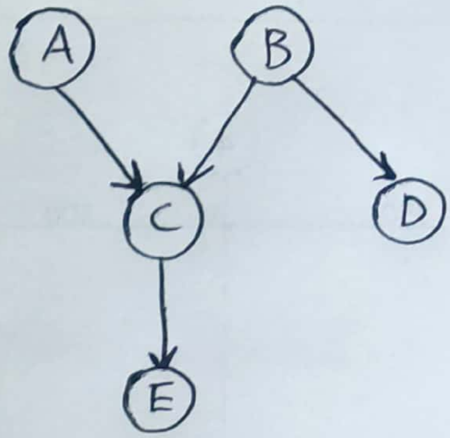
$\vdots$

$x_n^i \sim p(x_n | x_1^i, x_2^i, \dots, x_{n-1}^i)$

$\Rightarrow (x_1^i, x_2^i, \dots, x_n^i)$ is a sample from $p(x_1, x_2, \dots, x_n)$

$P(A,B,C,D,E)$

$= P(A)\,P(B)\,\underline{P(C|A,B)}\,P(D|B)\,P(E|C)$



$a^i \sim P(A)$

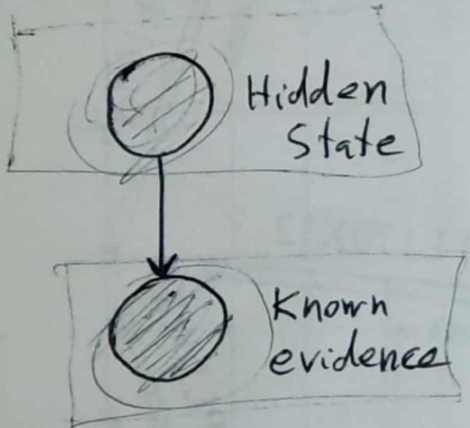
$b^i \sim P(B)$

$c^i \sim P(C \mid a^i, b^i)$

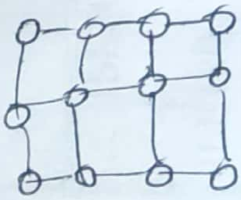
$d^i \sim P(D \mid b^i)$

$e^i \sim P(E \mid c^i)$

$(a^i, b^i, c^i, d^i, e^i)$ is a sample from $P(A,B,C,D,E)$



MRF $\quad P(X_1 - X_n) = \frac{1}{Z} \prod_{c \in C} \Phi_c(X_c)$



?

$P(A,B,C,D,E) = P(A)\,P(B)\,P(C|A,B)\,P(D|B)\,P(E|C)$

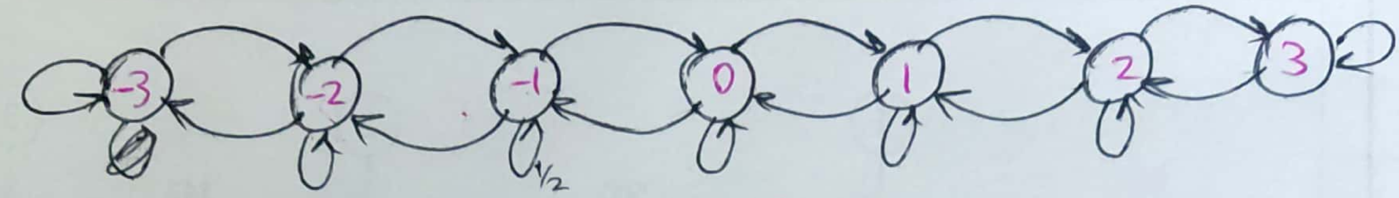
$\boxed{\begin{matrix} P(A \mid D,E) \\ P(A \mid D=d, E=e) \end{matrix}}$ $\Rightarrow$ solution 1: samples from $P(A,B,C,D,E)$

solution 2: samples from $P(A,B,C \mid d,e)$

$$P(A,B,C \mid d,e) = \frac{P(A)\,P(B)\,P(C|A,B)\,P(d|B)\,P(e|C)}{\sum_A \sum_B \sum_C P(A)\,P(B)\,P(C|A,B)\,P(d|B)\,P(e|C)}$$

$$= \frac{1}{Z} P(A)\,P(B)\,P(C|A,B)\,P(d|B)\,P(e|C)$$

$$= \frac{1}{Z} \Phi_1(A)\,\Phi_2(B)\,\Phi_3(A,B,C)\,\Phi_4(B)\,\Phi_5(C)$$



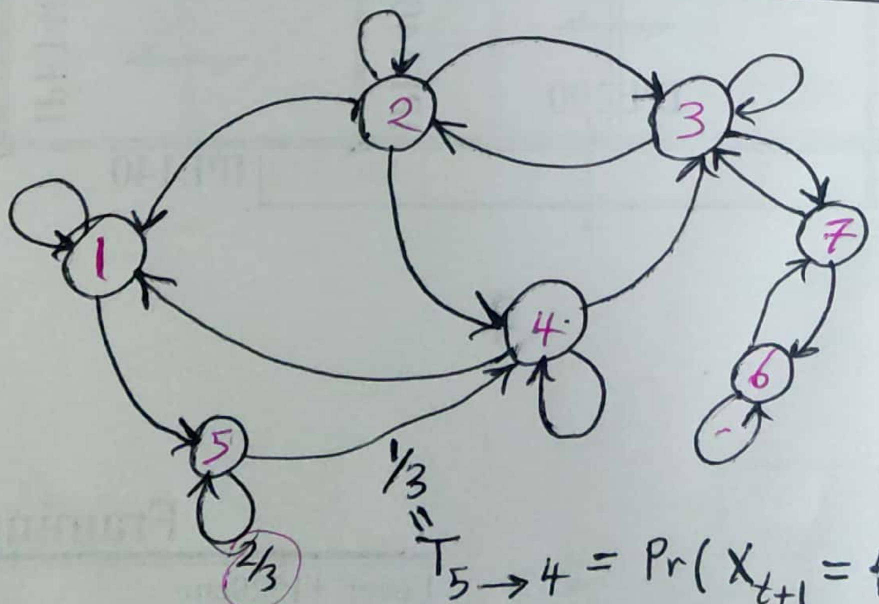
	-3	-2	-1	0	1	2	3
$P^{t=0}(X)$	0	0	0	1	0	0	0
$P^{t=1}(X)$	0	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0
$P^{t=2}(X)$	0	$\frac{1}{16}$	$\frac{1}{4}$	$3/8$	$\frac{1}{4}$	$\frac{1}{16}$	0
$P^{t=\infty}(X)$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

$\frac{1}{4} \times \frac{1}{4}$

$\frac{1}{4} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}$

$\frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4}$

$\frac{1}{4} + \frac{1}{16} + \frac{1}{16}$



$P(X)$

$X \in \{1,2,3,4,5,6,7\}$

X_t = ~~stat~~ state ~~curren~~ at time t

$\frac{1}{3} = T_{5\to4} = Pr(X_{t+1} = 4 \mid X_t = 5)$ independent of t

$= Pr(X_t = 4 \mid X_{t-1} = 5)$

$T_{5\to5}$

$$Pr(X_t = a) = \sum_{b=1}^{7} Pr(X_t = a \mid X_{t-1} = b)\ p(X_t = b)$$